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Noncommutative inflation and the large-scale damping in the CMB anisotropy*

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abstract

We show that a certain class of short-distance cutoff can give rise to large suppression on the CMB anisotropies at large angular scales.

1 Introduction

The inflationary models of the universe give a desired initial condition to the standard big bang cosmology. They are supported by the recent elaborate measurement of the CMB anisotropy [1], except that the observed angular power spectrum of the CMB for small multipoles l ($l \lesssim 10$) have much smaller values than the theoretical prediction. The standard explanation of this discrepancy at large angular scales is based on the cosmic variance [2]. However, if this is not simply a statistical deviation, then there is a possibility that this is a remnant of string dynamics in the very early universe.

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One of the basic results in string theory is the existence of the minimum length scale l_s [3], and spacetime is expected to loose its smooth Riemannian structure and to become discrete (or noncommutative) at the Planck scale. The main aim of this talk is to show that a short-distance cutoff or noncommutativity can suppress the CMB anisotropies at large angular scales [4].

This claim would be against one's intuition, because such short-distance structure usually does not give rise to important effects on large-scale physics in local quantum field theories. However, in the exponentially expanding universe as in inflationary models, a cutoff on comoving modes can be monotonically increasing function of time and, as we see below, can suppress large-scale modes when a particular class of cutoff is chosen.

Before proceeding to a discussion on this "large-scale damping," we here briefly review the basic ingredients in inflationary models.

The flat FRW metric is $ds^2 = -dt^2 + a^2(t)\, d\boldsymbol{x}^2$. By making a Fourier-transform of an inflaton field as $\phi(t,\boldsymbol{x}) = \sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\, \phi_{\boldsymbol{k}}(t)$, the physical wave length is given by multiplying the comoving wave length $1/k$ by the scale factor, and thus increases monotonically in the expanding universe:

$$\lambda_{\rm phys}(t) = \frac{a(t)}{k}. \tag{1}$$

On the other hand, the Hubble parameter is defined to be $H(t) \equiv \dot{a}(t)/a(t)$. During inflation $\big(a(t) = (1/H)\, e^{H(t-t_i)}$ (t_i: initial time of inflation)$\big)$, the Hubble length $1/H$ is constant, and thus for each mode $\phi_{\boldsymbol{k}}(t)$ there is the moment $t_{\boldsymbol{k}}^{\rm C}$ at which $\lambda_{\rm phys}(t_{\boldsymbol{k}}^{\rm C}) = 1/H$. Since the Hubble length gives the physical (proper) distance scale beyond which two points cannot be causally correlated, the mode $\phi_{\boldsymbol{k}}(t)$ becomes classical after crossing the Hubble horizon, $\lambda_{\rm phys}(t) \gtrsim 1/H$. By using the cosmological perturbation theory [2], the angular power spectrum of the CMB observed at the present time can be shown, at large angular scales, to be proportional to that of inflaton evaluated at the exit of inflation. In this sense, what we observe as the CMB anisotropies are the classical "fossils" of the quantum fluctuations of inflaton. Note that the smaller the comoving wave number k is, the earlier the corresponding mode crosses the horizon, and thus, the large-scale modes (with small k) become classical at early times during inflation.

2 A mechanism of the large-scale damping

Now we explain how a short-distance cutoff can affect the large-scale behavior [5].

In order to give a general discussion, we consider a mode expansion of the inflaton field $\phi(t,\boldsymbol{x})$ in a generic form:

$$\phi(t,\boldsymbol{x}) = \sum_A \big(a_A\,\psi_A(t,\boldsymbol{x}) + a_A^\dagger\,\psi_A^*(t,\boldsymbol{x})\big), \tag{2}$$

where $\{A\}$ is a set of comoving modes, and $\psi_A(t,\boldsymbol{x})$'s are symplectically orthonormal, positive-energy solutions to the Klein-Gordon equation on the curved spacetime, $ds^2 = -dt^2 + (1/H^2)\,e^{2H(t-t_i)}\,d\boldsymbol{x}^2$. This field is properly quantized by setting canonical commutation relations as $\big[a_A, a_B^\dagger\big] = \delta_{AB}$. We introduce a pseudo-order in the set $\{A\}$ such that small A implies a larger scale. An example is to simply set $A = \boldsymbol{k}$ and require that $A = \boldsymbol{k} < A' = \boldsymbol{k}'$ when $|\boldsymbol{k}| < |\boldsymbol{k}'|$. A mode A starts its quantum fluctuation at an early time, and becomes classical around the moment of crossing the horizon, which we set $t = t_A^{\mathrm{C}}$ (see Fig. 1 (a)).

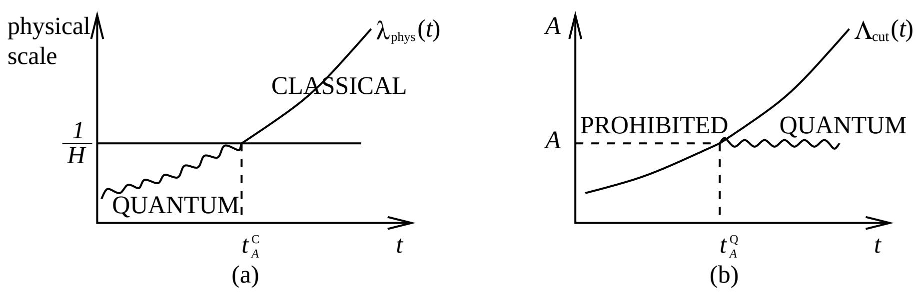

Figure 1: (a) t_A^{C} is the moment at which a mode A crosses the horizon and becomes classical. (b) t_A^{Q} is the moment when the mode A starts its quantum fluctuation.

On the other hand, in the presence of short-distance cutoff, the comoving modes have a cutoff on large A, $A \leq \Lambda_{\mathrm{cut}}$. As we see below, this cutoff Λ_{cut} can be a monotonically increasing function in time in the expanding universe, $\Lambda_{\mathrm{cut}} = \Lambda_{\mathrm{cut}}(t)$. Thus, to a given mode A one can assign the moment t_A^{Q} at which the mode A starts its quantum fluctuation (see Fig. 1 (b)). Before the moment ($t < t_A^{\mathrm{Q}}$) the mode A is prohibited to exist as a quantum fluctuation.

Suppose that there exists the following inequality with some mode A_c:

$$A \lesseqgtr A_c \iff t_A^{\mathrm{C}} \lesseqgtr t_A^{\mathrm{Q}}. \tag{3}$$

Then the modes $A < A_c$ and the modes $A > A_c$ behave quite differently:

(1) $A < A_c$ (larger-distance scale): In this case, the mode A must become classical before its quantum fluctuation starts, and thus the classical amplitude must be largely suppressed, because no fossil can exist if the mode does not have a life of quantum fluctuation. We thus expect that the corresponding temperature fluctuation in the CMB has large suppression.

(2) $A > A_c$ (shorter-distance scale): In this case, the mode A has a period of quantum fluctuation, leaving a classical fossil in finite size after crossing the horizon. Since the effect of cutoff is expected to disappear rapidly, the classical value will be almost the same with that in the case without cutoff. We thus expect that the corresponding temperature fluctuation has almost the same magnitude with that in the absence of cutoff.

Thus, with a cutoff satisfying the condition (3), the mean square of the classical amplitude will have a sharp damping on the large-scale modes $A < A_c$. (Actually, the process for a mode to become classical proceeds only gradually around the moment t_A^{C}, so that the damping will be smeared [5].)

3 Example – fuzzy sphere

We give a model of noncommutative inflation satisfying the condition (3). We here introduce the noncommutativity only to the angular coordinates [4] because their noncommutativity will be most relevant to the large-scale damping in the angular power spectrum of the CMB. In fact, in the cosmological perturbation theory, one can consider the time evolution of each mode separately. Furthermore, the angular power spectrum of the CMB anisotropy can be related to the fluctuations of gravitational potential on the last scattering surface, and thus is sensitive to the fluctuations only in the angular directions. Since the noncommutativity to a given direction is expected to give its major effects to the fluctuations in the corresponding direction, the introduction of noncommutativity to the other directions (time and radius) will not give a drastic change to the angular power spectrum.[a]

For later convenience, we introduce the conformal time η,

$$ds^2 = a^2(\eta)\big(-d\eta^2 + dr^2 + r^2 d\Omega^2\big), \tag{4}$$

[a]Recently Tsujikawa, Maartens and Brandenberger [6] and Huang and Li [7] have analyzed the noncommutative inflation introducing the noncommutativity only to time and radial coordinates, and shown that the effect is not strong enough to give a sharp damping at large angular scales.

for which the scale factor is expressed as

$$a(\eta) = -\frac{1}{H\eta} \quad (\eta < 0). \tag{5}$$

Note that η is negative during inflation, and the exit time of inflation is given by taking $\eta \to -0$.

We introduce a fuzzy sphere such that there are at most one-bit degrees of freedom in the physical area $L_{\rm cut}^2$. Since the physical area of the sphere for each (η, r) is $4\pi\big(a(\eta)r\big)^2$, the maximum degrees on the fuzzy sphere is given by [4]

$$\frac{4\pi\big(a(\eta)r\big)^2}{L_{\rm cut}^2} = \frac{4\pi r^2}{(L_{\rm cut}H)^2\eta^2} \equiv N(\eta, r) + 1, \tag{6}$$

which is actually a monotonically increasing function of time η (recall that $\eta < 0$). Since the mode expansion of a scalar field on the fuzzy sphere has a limiting multipole, $l \le \Lambda_{\rm cut} = N(\eta, r)$ [4], the inflaton field is expanded as

$$\phi(\eta, r, \Omega) = \sum_{l=0}^{N(\eta,r)} \sum_{m=-l}^{l} \int_0^\infty \frac{dk}{2\pi} \big(a_{klm}\, \psi_{klm}(\eta, r, \Omega) + \text{h.c.}\big), \tag{7}$$

$$\psi_{klm}(\eta, r, \Omega) = H\sqrt{\frac{2}{k}}\, \big(1 + ik\eta\big)\, e^{-ik\eta}\, j_l(kr)\, Y_{lm}(\Omega). \tag{8}$$

The moment $\eta_A^{\rm C}$ at which the mode $A = (k, l, m)$ becomes classical is obtained by setting $a(\eta_A^{\rm C})/k \equiv 1/H$:

$$\eta_A^{\rm C} = -\frac{1}{k}, \tag{9}$$

while the moment $\eta_A^{\rm Q}$ at which the mode starts its quantum fluctuation is given by setting $l \equiv N(\eta_A^{\rm Q}, r)$:

$$\eta_A^{\rm Q} = -\frac{1}{L_{\rm cut}H} \sqrt{\frac{4\pi}{l+1}}\, r. \tag{10}$$

The critical mode A_c can be roughly estimated by noting that $\eta_A^{\rm C}$ can be approximated by $-2r/\pi(l+1)$ because the spherical Bessel function $j_l(kr)$ has a sharp peak at $kr = (l+1)\pi/2$. Thus, setting $\eta_{A_c}^{\rm Q} = \eta_{A_c}^{\rm C}$, we have

$$l_c = \frac{(L_{\rm cut}H)^2}{\pi^3} - 1. \tag{11}$$

It is easy to see that the condition (3) holds with this l_c, and thus we expect that the angular power spectrum will have a sharp damping for $l \lesssim l_c$.

A more detailed analysis along the above argument is made in Refs. [4] and [5] with a constant spectral index n. For $n = 0.95$, the resulting damping factor is given in Fig. 2.

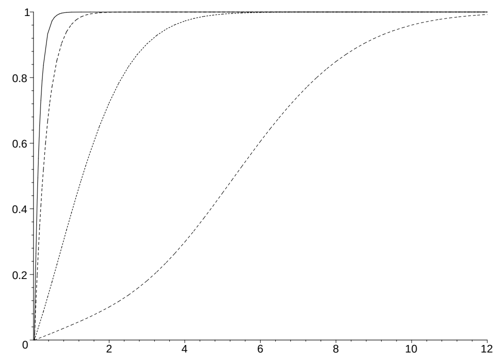

Figure 2: The resulting damping factor as a function of multipole l. Here the spectral index is set to be $n = 0.95$. $L_{\rm cut}H$ is 0.1, 1, 5, 10 from top to bottom [4].

4 Conclusion and outlook

We have shown that a class of short-distance cutoff (or noncommutativity) can give rise to suppression on large-scale modes. In order to have a damping for $l \lesssim 10$, the noncommutative scale $L_{\rm cut}$ is of the same order to the Hubble scale $1/H$. In Ref. [5], the model using a fuzzy sphere is further investigated, by setting the initial condition on the inflaton fluctuations such that every mode starts its quantum fluctuation as the vacuum fluctuation with respect to the Hamiltonian upon being released from the constraint of cutoff. We also sketch a holographic rationale for the cutoff using a fuzzy sphere. An investigation along this direction is now in progress and will be reported elsewhere.

Acknowledgments

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